Soil-Structure-Interaction Effect on Seismic Response of Piles at Bridge Pier Supports

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ABSTRACT

Efforts have been made in the past to investigate and quantify the behaviour of short length piles subjected to lateral loading such as that occurring during earthquakes. These studies were motivated from field observations where it was found that damage could be mitigated in strong ground motion events when the structure is supported on compliant, shallow pile foundations. The effectiveness of this mechanism of behavior is owing to the reduction of rotation demand in the piers due to the capacity of the pile-head to rotate thereby participating in the deformation response, through a "spring-in-series" arrangement with the pier (i.e. $K_{pier} = K_{piles}+K_{column}$). Some benefit to design can be obtained from this behavior if a dependable estimation of the equivalent foundation stiffness can be established as a function of rotation demand. Modeling the foundation as a rigid pile head supported by a group of piles, it is the objective of this paper to study its characteristic resistance curve; in this regard several pushover analyses of a single pile under kinematic constraint at the point of connection with the overbearing pile-head were conducted, in order to observe the intensity and influence of the soil-pile interaction on the superstructure deformation demands when excited by ground motion events of increasing intensity. Parametric investigation is conducted to support the development of quantitative models for the coordinates of the milestone points of the resistance curve in terms of the important design variables (soil profile, pile aspect ratio, etc.) which could then be scaled up to integrate into the resistance curve of a complete pile-group. In this study an example bridge overpass for an Ontario Highway is considered in relevance to the Canadian practice.

Keywords: Lateral Loading, Pushover Curve, Pilecap, Pier, Pile Foundation

INTRODUCTION

In recent years several studies reported on the effect of soil structure interaction on the response of superstructures such as buildings and bridges to static and/or dynamic ground excitation [1], [2], and some design recommendations have been developed in this regard [3]. The interaction between the foundation (substructure) with the surrounding soil impacts two important dynamic characteristics of the system, namely the effective stiffness [4] and damping [5]. The implication of stiffness modification resulting from the foundation compliance is to reduce the seismic demands from the vulnerable components of the superstructure (e.g. piers). This concept has attracted attention from researchers as it presents an opportunity in bridge design to limit the design requirements for pier detailing at the ultimate limit state. Relevant research and illustration of concepts is through back analysis of field records from instrumented existing bridges (with deep or shallow foundations) [6].

In addition to the research on field data, some conceptual approaches have been attempted in order to formulate methods by which the beneficial aspects of Soil-Structure-Interaction (SSI) may be explicitly accounted for in design to lead to an improved structural system. For example, considering the embankment participation in monolithic bridges, where the deck is driven by the embankment displacements, a bent with stiff foundation undergoes the total deck displacement demand through flexural deformation and commensurate damage; improved pier performance may be envisioned by effecting stiffness reduction of the individual bent columns through a simultaneous increase in their number and reduced sectional size of columns per bent, so as to obtain increased deformation capacity. Another alternative to achieve reduced pier damage and to exacerbate the impact of the pile contribution is to secure increasing foundation compliance through deformation of the piles, preferably with increasing ground motion intensity. Several parameters define and govern the extent of benefit or detriment effected by SSI on pile-foundation. Objective of the present work is to study the sensitivity of the problem to its important design variables, such as the type of soil media (surrounding the foundation), the depth of the pile foundation, type of foundation (short or deep piles), and the overbearing superstructure forces that are transferred as axial forces on the foundation. The resistance of piles to a pilecap transverse translation or rotation at the point of connection was calculated for increasing magnitude of kinematic disturbance under displacement control and is represented herein in the form of pile pushover resistance envelopes. The problem was modelled using a 2D finite difference program (L-Pile) [7]. The reference structural model studied (pilecap interacting with individual pile and soil) is part of an actual overpass bridge on Hwy 401 (Fig. 1), located in East Ontario. The original bent foundation system consists of eighteen driven H-Piles that are arranged in two groups of nine (one group per bent column), all connected to a massive pilecap and driven till bedrock. As mentioned before, this paper will focus on response of a pilecap-pile system subjected to an incrementally increasing horizontal translation and a rotation imposed at the centroid of a massive (rigid) pilecap. Values are modified using reduction factors to account for the interaction of piles [8], [9] in order to establish a complete foundation stiffness under the bridge bents. The case study considered consists of a two span continuous, 74m long deck, supported through bearings on abutments and a bent cap. The bent is a planar frame comprising two columns, which are anchored into the pilecap; reinforcement extends into the footing by 0.6m. To enable the sensitivity analysis, piles are extended to various depths above bedrock so as to model various degrees of compliance in the boundary conditions of the pile object, whereby the pile tip (free end of the pile) can displace and/or rotate.

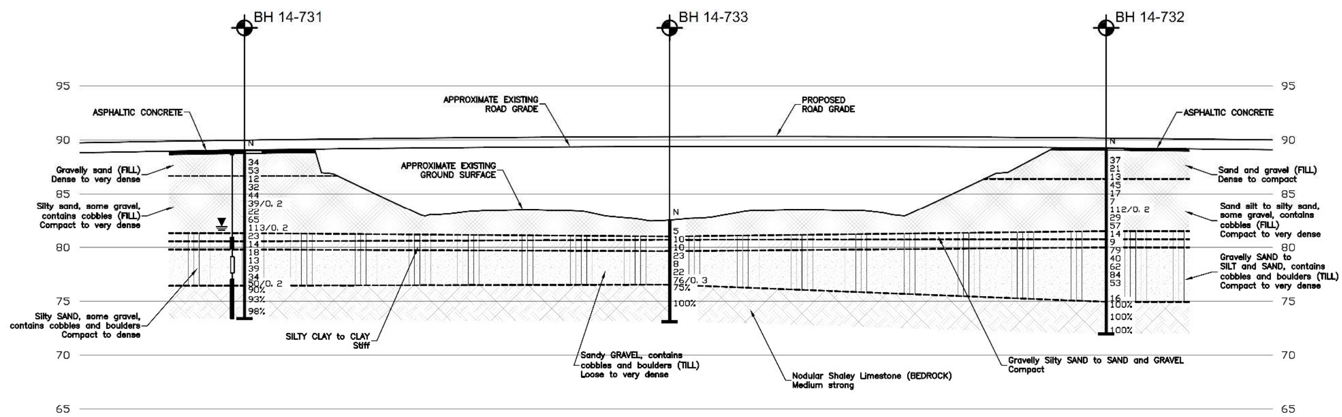

Figure 1. Reference Overpass on Hwy 401 – Ontario [10]

As illustrated in Figure 1, soil data are available from three boreholes that had been obtained in the context of site investigation by a geotechnical consultant [10]. Different types of soil are found in the provided report. For consistency of the results and to facilitate generalization of the findings, two soil types, namely a stiff clay and a loose sand were adopted in the models considered, in order to examine a broad range of possible values in the ground properties. First, stiff clay was considered as the embedment layer. In consecutive soil models, a sand layer gradually replaced clay, increasing in thickness from pile-tip to the top of the pile-cap, in 3 increments, in order to observe the effect on the system's response to the above-mentioned imposed pile-head displacements.

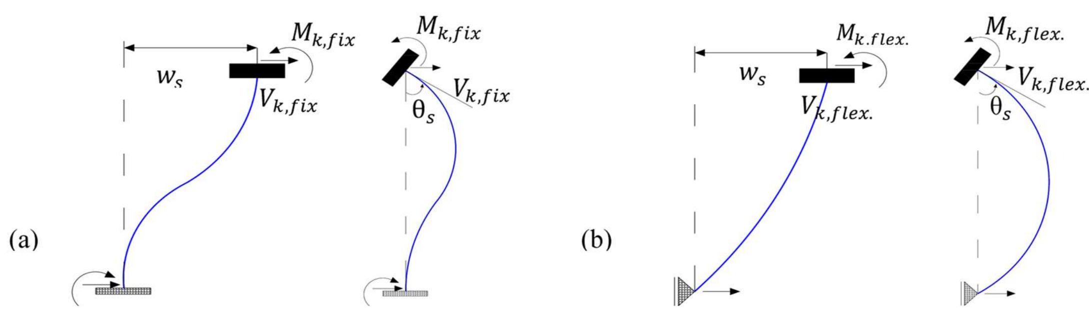

Figure 2. Pile Head Imposed Displacements (a) Clamping in bedrock, (b) Compliant pile (not driven to bedrock)

The bridge replaced an older structure that had different foundation elements in terms of foundation material and pile diameter, old piles were treated as a benchmark point, set to be modelled and contrasted with the replaced piles, which will be reflected here as a part of the evaluation of results. Moreover, the axial force bearing on the pile, representing the self-weight of superstructure was varied in the sensitivity analysis considering its effect on foundation pushover resistance. Finally, pile length was varied, with the maximum length extending down to bedrock. The latter case serves as a point of reference in the pushover analysis, where the pile tip may be considered either free to translate in soil, or to be clamped in bedrock, thereby impacting the effective stiffness and the magnitude of forces attracted at the pile head in response to the imposed top displacements (Fig. 2). For the problem studied it is assumed that the pilecap experiences a lateral translation w_s driving along all the piles by the same lateral displacement; lateral resistance to this translation is mounted by the soil surrounding the pile, however the relative compliances of soil and pile play an important role here. In this regard, the stiffness of the pile, its support conditions at the tip, the length, the soil modulus and the pile elastic modulus are all critical parameters. Figure 2(a) represents the case of a restrained pile tip (non-rotational and non-translational fixity), whereas the two degree of freedom at the top node of the pile (lateral translation, w_s, and chord rotation, θ_s) are given gradually increasing values so as to obtain a pushover resistance curve. In Fig. 2(b) the opposite case is depicted, where the pile tip does not extend to bedrock and therefore that point is free of stress (zero moment and zero shear). For the elastic case [11] the case with fixed tip develops twice as large bending moment and four times larger shear force reaction to rotation or translation respectively at the connection point with the pilecap as compared to the non-fixed tip case.

$$\begin{Bmatrix} M_{k,fix} \\ V_{k,fix} \end{Bmatrix} = \frac{2EI}{l}\begin{bmatrix} 3 & 2 \\ 6/l & 3/l \end{bmatrix} \cdot \begin{Bmatrix} w_s/l \\ \theta_s \end{Bmatrix} = \begin{pmatrix} \frac{6EI}{l^2}.w_s + \frac{4EI}{l}.\theta_s \\ \frac{12EI}{l^3}.w_s + \frac{6EI}{l^2}.\theta_s \end{pmatrix} \tag{1}$$

$$\begin{Bmatrix} M_{k,fix} \\ V_{k,fix} \end{Bmatrix} = \frac{3EI}{l}\begin{bmatrix} 1 & 1 \\ 1/l & 1/l \end{bmatrix} \cdot \begin{Bmatrix} w_s/l \\ \theta_s \end{Bmatrix} = \begin{pmatrix} \frac{3EI}{l^2}.w_s + \frac{3EI}{l}.\theta_s \\ \frac{3EI}{l^3}.w_s + \frac{3EI}{l^2}.\theta_s \end{pmatrix} \tag{2}$$

Equations (1) & (2), qualitatively estimate the force reactions on the pile head in response to horizontal displacement; these forces are transferred to the superstructure bents (E is the modulus of elasticity of pile, l is the pile length). For an inelastic pile embedded in a nonlinear soil environment the stiffness of the pile is a function of the imposed displacement. To determine this relationship numerically the pushover analysis is conducted for the pile-soil assembly under increasing displacement at one of the degrees of freedom (d.o.f.) at the pile head each time (Fig. 3). To isolate the effect on the individual stiffness terms all other d.o.f. are restrained when conducting the pushover analysis for one of them on the pile-soil system. Results are presented in the following section; parametric studies include the pushover curves and the corresponding evolution of the secant stiffness contributions associated with the translational and rocking degrees of freedom at the top of the pile, K_{11}, K_{22}, and K_{12} (here this is taken equal to K_{21}. Based on the analysis results with actual soil data it was found that this equality is only an adequate approximation). These values are intended for modeling the interaction between soil and superstructure in modeling the lateral response of complete bridges and depend on the important parameters of the problem (ground type, pile slope, length, pile dimension and shape.)

SSI EFFECT ON COMPLIANT BASE STRUCTURES

Mechanics of SSI Participation in Superstructure Response

In the earlier sections the advantages of partial contribution of the pile foundation to stiffness of the Bridge system were considered as a vehicle to reduce the seismic demand on piers [12]. This concept was understood after evaluation of field data in instrumented bridges in the US (e.g. Painter street overcrossing, California), illustrating that damage to the bridge bents was mitigated owing to a rotational compliance at the pier supports during moderate seismic events. Recently the number of instrumented bridges has increased on account of this observation, while several analytical and numerical studies have been used to correlate the field records with the beneficial effect of SSI on short pile foundation bridges. Initially the emphasis was placed on the abutments, since it was determined that due to their being supported on short pile foundations, they are driven by the embankment response during earthquake motion carrying along the bridge deck in rigid-body translation to the same displacements. In monolithic structures this renders the piers the most vulnerable component of the bridge, owing to the excessive drift demands; significant pier damage is expected in that case if the pier is stiff and the foundation non-compliant. But a compliant foundation, in such cases, relieves partly the drift demands, since part of the drift is developed inside the foundation, by translation and rotation of the pilecap, through deformation of the piles [13].

The right side of Figure 3 depicts the base of the pier of shown to the left, with the definition of the degrees of freedom considered. The effective translational stiffness of the bent, is modified from the reference value associated with pier deformation alone ($K_{pier} = 12EI/H^3$), when soil is compliant, as follows: If Φ is the fundamental mode shape of translational vibration of the pier-foundation system, and it includes nonzero share of deformation below grade as depicted in the figure, then the total effective stiffness (from Virtual Work) is:

$$\boldsymbol{K^{eq}_{pier-foundation}} = K_{pier} \,.\, \Delta\phi^2_{pier} + K^{transl.}_{found}.\,\Delta\phi^2_{found} + K^{rotat}_{found}.\,(\frac{\partial\phi}{\partial z}|_{z=H_B})^2 \tag{3}$$

where the first term represents work done through pier deformation, the second through translation of the foundation and the third represents work done by rocking of the pilecap (here $\Delta\Phi_{found}=\theta_s \cdot H_B$). In the present analysis, nonlinear p-y curves are used to calculate the response of horizontal springs attached to the pile skin as the pile undergoes lateral translation or head rotation; the governing equations are solved using a finite-differences equation [14]. The approach is based on Winkler's original method of subgrade reaction [15]. Two types of ground, stiff clay and loose sand are used in the pile pushover analysis; results are plotted in the form of shear displacement (V-Δ) and moment rotation (M-θ) curves; secant values for the stiffness terms that couple the d.o.f and the corresponding actions shown in Fig. (3b) are also given in plots against the imposed displacement / rotation magnitude. Results from the analysis can be used to develop resultant foundation springs at the base of a F.E. bridge superstructure model to represent the overall nonlinear foundation compliance [16], [17], [18], without the need for detailed modeling of the substructure.

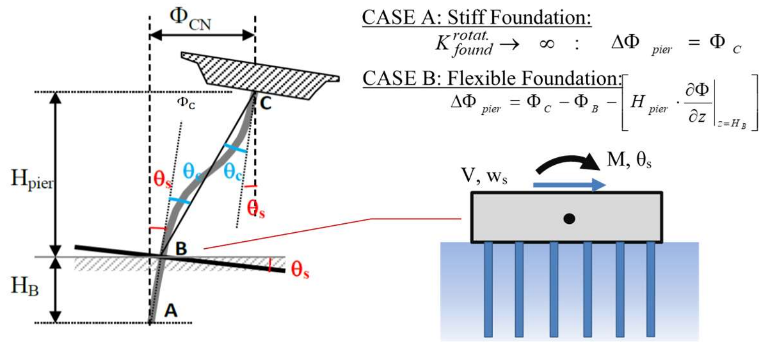

Figure 3. Deck-Pier-Foundation connection: (a) Definition of terms, (b) Degrees of freedom at pilecap. Note that demands in individual subcomponents are partly mitigated if all such elements of the system are engaged in deformation.

Soil properties taken from the geotechnical study of the reference bridge were as follows: Stiff Clay with unit weight of 19.3 kN/m^3 with a shear strength of 73 kPa. Loose Sand with unit weight of 18.9 kN/m^3 with a friction angle of 31.9^o. For the purpose of analysis, the Reese models for Clay and Loose Sand were selected in L-Pile. At the bottom of the compliant ground layers, there is a thick layer of bedrock. Thickness of all layers of soft soil extends down to 12 m. Pile Length is varied to study its effect, but in the free tip case (where the pile tip does not penetrate in bedrock) the length of pile is assumed to be 3.5 m. Thickness of pile cap is 1.2 m with 0.75 m below the ground surface. The typical pile is fixed in the pilecap. In all cases with the exception where the pile section size is varied, pile is considered to be an H-310×110 steel pile. Values of theoretical strength in shear (V_r), in flexure (M_r), as well as the elastic stiffness coefficients corresponding to the pile with this cross section with l=3.5m, are given as horizontal boundary lines in the plots (note that moments are listed with reference to the secondary axis on the right side of each plot, whereas shear is listed on the primary axis in the left hand side.) Where the theoretical values are well above the calculated ones, it is clear that the strength of the system is controlled by the soil; this is the case of the majority of the cases considered herein, underscoring the necessity to account for the soil compliance in structural models of this type.

RESULTS AND DISCUSSION

Effect of Axial Thrust on Foundation Response to Pile Cap Deflection and Rotation

In this section pile and pilecap are fully embedded in stiff clay. Axial loads of 200kN, 1000kN and 2000kN, were applied to the pilecap-pile system. Fig. (4) presents the effective shear (reference to the principal axis) and effective moment (reference to the secondary axis to the right of each plot) that is applied by the pilecap on the pile in order to produce the required displacement or rotation (left and right plots, respectively, coordinate in the horizontal axis). It is also observed that higher axial loads expedite failure of the pile under lateral displacement or pilecap rotation. Secant stiffness of the pile under 1000kN and 2000kN thrust is almost the same, within small displacements (up to 10mm) and rotation (up to 0.14 rad) of pile cap. It is evident that under lower axial force, yielding of the pile-soil system happens at a higher pile cap shear and moment reaction, and practically, in this case, failure never happens. Early strength loss of the piles under higher axial pressure can be related to buckling of the pile and the eccentricity effect (P-Δ) after either local yielding in the pile or ground failure at one side of the pile. For equivalent stiffness under the three different axial loads, it is observed in Fig. (5) that there is a consistency between three curves, implying a gradual increase of stiffness (in all terms, K_{11}, K_{12}, K_{21}, K_{22}) with increasing axial load. This shows that the case under lower thrust (here 200 kN), is more resilient under applied lateral displacement and pile cap rotation. Moreover, it confirms that the foundation under higher vertical loads transfers higher pile head forces to the superstructure, but only in the low range of displacement and rotation demands. The pile carrying 2000kN thrust fails after a few increments in pile cap deflection and rotation, due to loss of stiffness, which leads to collapse of the whole system. It is important to note that the effective stiffness of the pile against rotation is negligible as compared to the static stiffness of the isolated pile element.

Effect of Pile Length on Foundation Response to Pile Cap Deflection and Rotation

In this section of the investigation the length of pile is the study variable. For this reason, its value is extended from 3.5m to 4.8m and 10m. Of the three cases the first instance represents a compliant pile, second and third examples, however, serve as fixed based foundation, whereby pile tip is clamped in bedrock. Axial thrust is 1000 kN and ground solely consists of stiff clay.

Figure 6 plots, with reference to the principal axis shear force (solid lines, each corresponding to a different length of pile), and with reference to the secondary axis the flexural moment exerted by the pile cap on the pile head (dashed lines, again, each corresponding to a different pile length); the respective reference strength values obtained from the pile cross section alone are

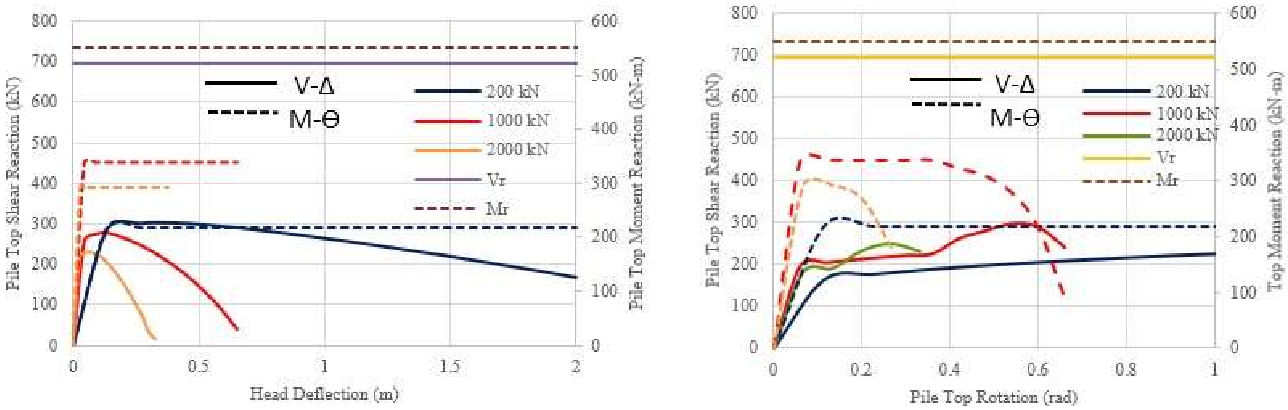

Figure 4. Pile Push-Over curves – Effect of increase in Thrust magnitude on foundation response

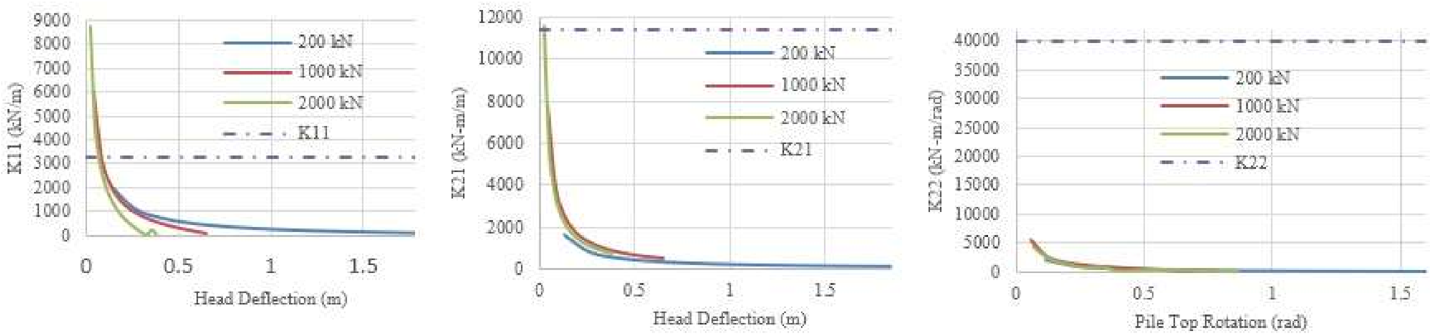

Figure 5. Stiffness curves – Effect of increase in Thrust magnitude on foundation response

given by the horizontal lines (solid for V_r and dashed for M_r). From Fig. (6) it is seen that the compliant foundation has larger plastic deformation compared to the piles with fixed head in bedrock. It is necessary to mention that the program didn't converge for displacement values exceeding 180 mm in the second and third scenario. Besides, the piles with rigid end bearing carry higher forces (bending moment and shear on pile top, as per Eqn. (1)) at the same rotational drift ratio, as compared to the compliant pile. This implies that a much higher secant stiffness is available within the elastic deformation range for the second and third case. Furthermore, there is a complete agreement between the stiffness values of cases two and three indicating a much-reduced SSI effect as compared to the first scenario. In Fig. (7) the evolution of secant stiffness curves obtained by varying the displacement for the three values of pile length is depicted. It appears that a single backbone curve may be used to describe the pattern of each of the stiffness diagrams. Therefore, the evolution of the system's effective stiffness is uniquely defined with the intensity of displacement only. This means that here the soil property controls the total pile-soil effective stiffness. Same is true for the strength values. However, more compliant foundation develops lower forces at the pile head, leading to reduced soil damage and reduced demands on the overbearing bridge pier. As before, capacity is exhausted first (shear) in the shorter pile driven to bedrock (4.8 m) on account of the higher shear demand at a given displacement level.

Effect of Pile section on Foundation Response to Pile Cap Deflection and Rotation

The effect of pile section stiffness was also considered. Two different H-pile sections are studied. A third case is generated to observe the effect of pile cap loading direction. This will lead to two different cases in which an H-pile connected to a pilecap, one bent in the direction of its weak axis and once in direction of its strong axis. Again, in Fig. (8) push-over curves of the cases specified above are plotted. In first case an H-Pile 310×110 is available, which is subjected to lateral displacement and rotational drift along its strong axis. The second scheme concerns an H-Pile BP 10×42 with same length as the first and third case (all 3.5m). Here, all three foundations are modelled in stiff clay. Again, in the current examples, foundations that developed higher shear reaction on top, due to pile cap rotation or deflection, suggesting the tendency to advance higher top bending moment reaction in response to imposed displacements. For instance, HP 310×110 that was shifted along its strong axis, shows twice as large bending moment reaction on top as compared to BP 10, which lines up with the plastic moment capacities of piles, nonetheless the foundation top shear reaction by HP 310 along strong axis is only slightly higher than that of BP 10.

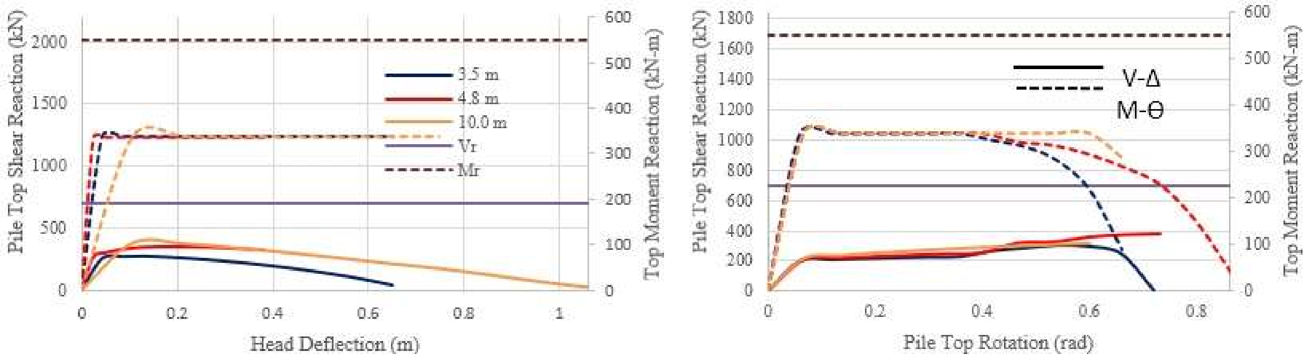

Figure 6. Pile Push-Over curves – Effect of increase of Pile Length on foundation response

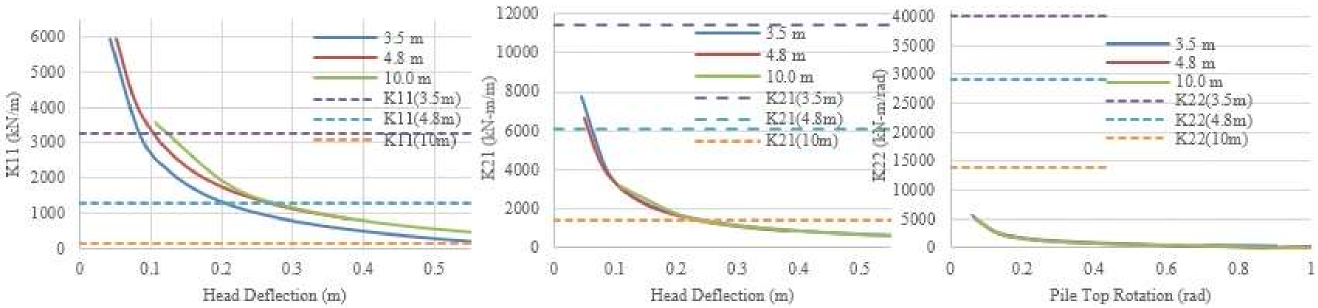

Figure 7. Stiffness curves – Effect of increase of Pile Length on foundation response

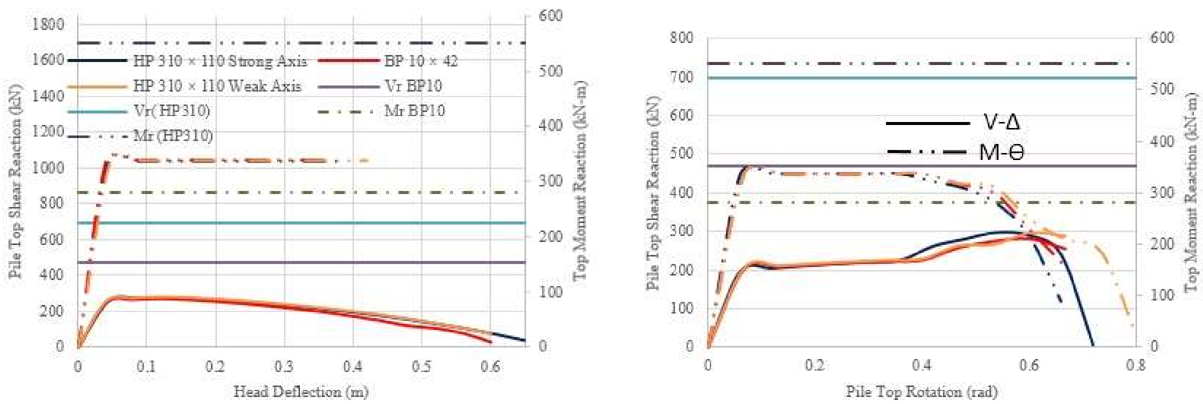

Figure 8. Pile Push-Over curves – Effect of change in Pile section on foundation response

To be reminded that the moment of Inertia of the HP 310 along strong axis is almost 2.7 the value for BP 10 and HP 310, subjected to deflection along its weak axis. Figure 9 shows good agreement in stiffness plots of all three samples. Therefore, the effect of steel pile section on horizontal stiffness of the foundation is almost negligible. However, slight discrepancies are noted. HP 310 pile underwent higher rotation or lateral displacement compared to the other two cases. In regards to rotational stiffness, it is noted that in second and third examples, with lower plastic moment capacity and smaller moment of Inertia, follow almost the same horizontal and rotational stiffness pattern effectively controlled by the soil performance.

Effect of Soil Type on Foundation Response to Pile Cap Deflection and Rotation

In this section the effect of soil layer on response of foundation to pile cap deflection is investigated. Five different cases were modeled. The first case complete pile is embedded in stiff clay. In the second example the foundation is completely embedded in loose sand. Third, fourth, and fifth examples are meant to understand the foundations in a combined stiff clay - loose sand ground medium. In the third case, the bottom half of pile is surrounded by loose sand, while the pile head and cap are embedded in stiff clay. In the fourth instance, ¾ of bottom of the pile is in loose sand and ¼ of top of the pile and pile cap are in stiff clay. In the last case, the pile is fully embedded in loose sand and the pile cap is in stiff clay. The pile length is 3.5 m and not clamped in bedrock (free tip). Applied thrust in all 5 cases was 1000 kN. Considering Fig. (10), the last two illustrations with foundation embedded in combined stiff clay and loose sand indicate higher shear and bending moment reaction at pile head. Meanwhile, having the foundation enclosed by stiff clay, a lower reaction on the pile top for smaller magnitude of pile cap deflection is observed. The case with pure loose sand follows the last two cases, whereby the pile head response for small increments of lateral shifting and pile cap rotation is lower than them, but higher than the response in pure stiff clay.

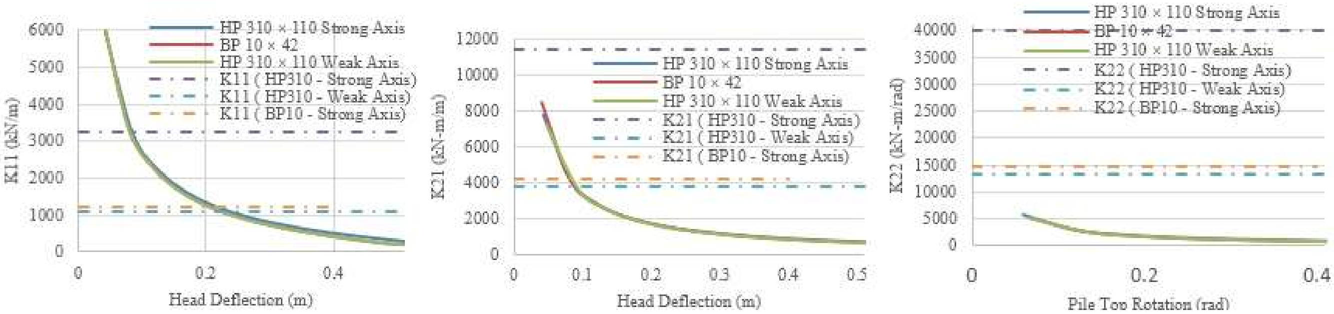

Figure 9. Stiffness curves – Effect of change in Pile section on foundation response

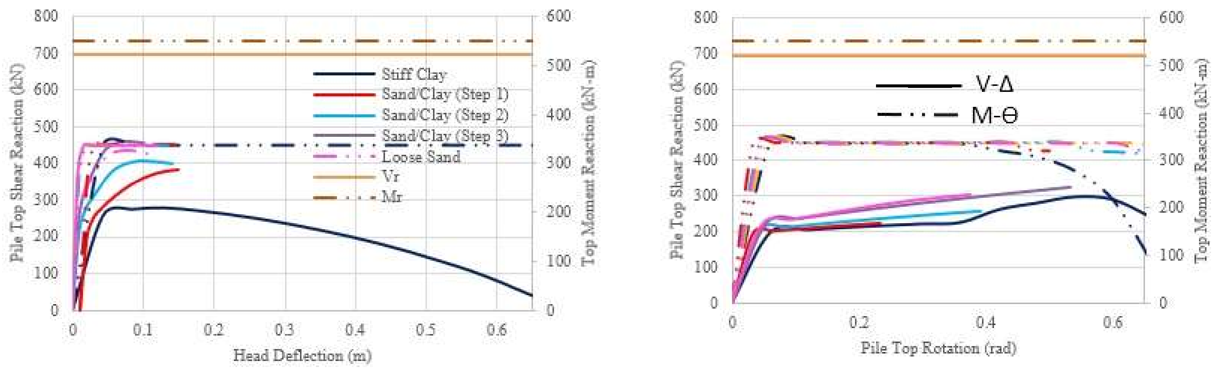

Figure 10. Pile Push-Over curves – Effect of change in Soil Type on foundation response

The pile enclosed over half its length with loose sand has the lowest pile top reaction in this range. Moving forward to larger displacement, for the last two cases (mixed soil, ¾ of pile length loose sand, and full pile length loose Sand), and pile completely embedded in loose sand, by lateral movements longer than 100mm the program fails to converge for the value of pile head shear reaction. Convergence limit for pile top bending moment reaction is as low as 80mm. Also, for the noted items, the program does not converge for pile cap rotation larger than 0.1 rad. On the other hand, from Fig. (11) it is understood that the stiffness diagrams for all four components follow a unique hyperbolic pattern that could be generalized for many cases, which is of interest of this study. For all that, within the range of 100mm to 300mm of pile cap lateral movement, and also for 0.15 to 0.3 rad of pile cap rotation, there are sort of disagreements in stiffness of the proposed foundations in discrete soil types. This effect is more remarkable for lateral stiffness of the substructure.

CONCLUSIONS

In the current practice, piles are preferably designed to penetrate an incompressible ground, to restrain pile tip in all degrees of freedom. However, this practice, to mitigate any possible damage and strength loss to the foundation imparts higher damage to bridge bents during seismic events. It was illustrated that in case of high intensity ground motions, it may be beneficial that foundation piles contribute to lateral stiffness of the bents, so that the bent columns are relieved from extensive damage at the expense of some acceptable damage in the foundation. The basis of this design approach is the structural mechanics principle, that effective stiffness of a structural system is controlled by the least stiff component that participates in energy absorption though deformation. Practically, this could be achieved by a compliant substructure. Explicitly in case of a pile foundation, pile tips should be embedded in soil layers, where they can displace and/or rotate. For different thrust values, vertically acting on pile top, lower thrust corresponds to higher pile top strength and therefore stiffness, whereas for small lateral displacement and drift, the pile under lower thrust magnitude release lower resistance at the top. The latter case shows larger flexibility under displacement and rotation compared to the first two. As expected, longer piles, with tip in the bedrock, have higher stiffness, and their resistance is larger than that of a compliant pile of similar length but with the tip in stiff clay soil. It is noted that once the pile is fixed in bedrock, the length of penetration barely affects the stiffness of the system, and the pile head reaction forces. Pile section analyses illustrated the fact that a pile with higher moment of inertia and plastic moment capacity, is less compliant. This higher stiffness suggests higher top reaction forces. However, it was concluded that change of pile section does not have a profound effect on the effective pile lateral stiffness due to rotation or top deflection, contrary to the isolated pile stiffness (non-interacting with soil). It was shown that the response of the system is more complicated when the soil layer differed gradually from stiff Clay to loose Sand over the pile length. It seems that piles embedded in pure Clay or half-length in Clay and the other in Sand have the lowest stiffness. This can be related to early failure development at the Sand Clay interface. On the other hand, the effective stiffness of the pile-soil systems, where the Sand layer gradually replaces the Stiff Clay, upwards (above the critical half length of the pile), increased and higher pile reactions were seen. Stiffness evolution in all cases examined could be described in a generalized manner with the same curve pattern, suggesting that they are all affected in the same manner by the degrading soil stiffness (for increasing displacement or rotation). It is concluded that potentially lateral or rotational stiffness of pile can be estimated parametrically with any combination of stiff Clay and loose Sand.

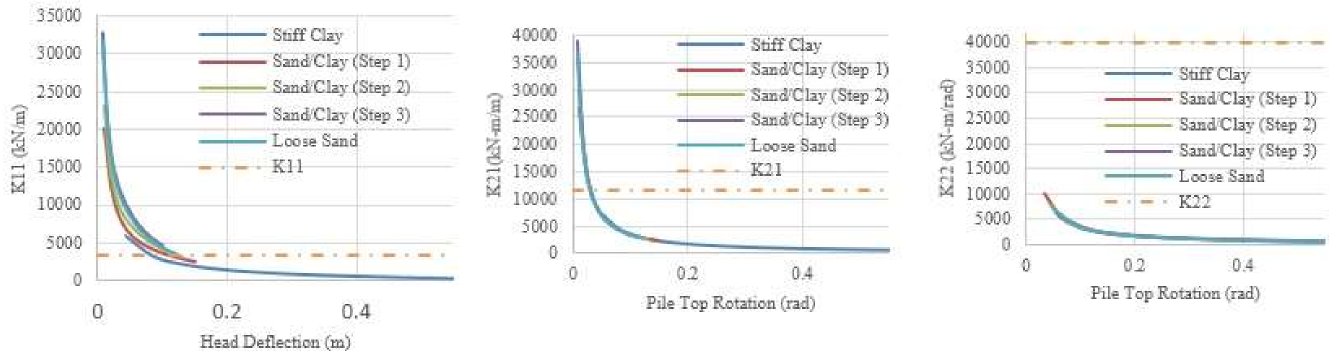

Figure 11. Stiffness curves – Effect of change in Soil Type on foundation response

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